Test 2 Mechanics & Relativity 2018-2019

Thursday October 11, 2018, 9:00 – 11:00, Aletta Jacobshal

Before you start, read the following:
There are 4 problems for a total of 45 points
Write your name and student number on each sheet of paper
Do no separate the exam-strack and try to fit all answers on them
Two spare sheet are added at the back of the stack
Make clear arguments and derivations and use correct notation
Support your arguments by clear drawings where appropriate
Write in a readable manner, illegible handwriting will not be graded

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Problem 1	**: ………. points out of 13**
Problem 2	**: ………. points out of 12**
Problem 3	**: ………. points out of 10**
Problem 4	**: ………. points out of 10**
Total	**: ………. points out of 45**

GRADE = 1 + #points/5 =

Lorentz Transformation equations, with $\gamma \equiv 1/\sqrt{1-\beta^2}$

$$\begin{aligned} t' &= \gamma(t-\beta x)\ , \qquad t = \gamma(t'+\beta x')\ , \\ x' &= \gamma(x-\beta t)\ , \qquad x = \gamma(x'+\beta t')\ , \\ y' &= y\ , \\ z' &= z\ . \end{aligned}$$

Einstein velocity transformations

$$\begin{aligned} v'_x &= \frac{v_x-\beta}{1-\beta v_x}\ , \qquad v'_y = \frac{v_y\sqrt{1-\beta^2}}{1-\beta v_x}\ , \qquad v'_z = \frac{v_z\sqrt{1-\beta^2}}{1-\beta v_x}\ , \\ v_x &= \frac{v'_x+\beta}{1+\beta v'_x}\ , \qquad v_y = \frac{v'_y\sqrt{1-\beta^2}}{1+\beta v'_x}\ , \qquad v_z = \frac{v'_z\sqrt{1-\beta^2}}{1+\beta v'_x}\ . \end{aligned}$$

NAME: ……………………………..……………………………..……………………………
STUDENT NUMBER: s……………..……………………………...……………………………

Problem 1 – Muons-II (13 points)

In the g-2 experiment fast muons are stored in a ring with a magnetic field which makes them fly around in a circle. The resulting acceleration does not play a role in answering the questions.

(a) The radius of the circular path of the muons is 7112 mm. What is the circumference in SR units? (**1 point**) **[1/2 point if only correct formula, 1 point for correct numerical result]**

$C = 2.\pi.R/c = 2*3.1412*(7.112\text{m})/(3\text{x}10^8 \text{ m/s}) = 148.9$ ns **[150ns or more precise OK]**

(b) The gamma-factor of the muons is 2.93. In the lab-frame, how long does it take a muon to make one full turn in the ring? Give a <u>formula</u> and <u>numerical result</u>. (**2 points**)

$\Delta t = C/\beta = 158.4$ ns ; $\beta=(1-1/\gamma^2)^{1/2} = 0.940$ **[-1 pnt wrong nr/no formula]**

(c) Inside the ring curved plates are placed to steer the muons. In the muon frame the length of these plates (along the line of motion of the muons) is 1.19 m and their height 50 mm. Calculate the length and height of the plates in the lab frame. (**3 points**) **[SR UNITS OK]**

$L = L' * \gamma = 1.19 \text{ m} * 2.93 = 3.48$ m **[no unit=0pnt, 1pnt if only formula, 1½ if nr OK]** $H= H' = 50$ mm **[no unit=0pnt, 1pnt if only H=H', 1 ½ point if correct nr]**

(d) In the lab-frame, muons are found to decay with a lifetime of 6.44 μs. Calculate their rest-frame lifetime. (**1 point**)

$\Delta t = \Delta t' / \gamma = 6.22\ \mu s / 2.93 = 2.198\ \mu s$ **[incorrect nr+correct formula = 1/2 pnt]**

(e) When a muon decays, some times an electron is emitted in the *backwards* direction (in the muon frame). If the emitted electron has speed |**v'**|=0.5 in the muon frame, what is its *speed* in the lab-frame? (**2 points**)

Use Einstein velocity transformation formula. We have $\beta=0.94$ from (b). Further we have $v'x=-0.5$, $v'y=0$, $v'z=0$. Readily, we have $vy=vz=0$. Use formula sheet to find $vx= (v'x+\beta)/(1+\beta.vx)=(-0.5+0.94)/(1-0.5*0.94)=0.83$. **Wrong sign for $\beta \rightarrow vx= 0.98$** Speed = $\lvert v \rvert = vx = 0.83$. **Missing $\lvert v \rvert$: –1/2pnt; wrong v'x sign: –1pnt; $v=v'x+\beta$: 0pnt**

(f) At other times, the electron is emitted *sideways*. If the electron has again |**v'**| = 0.5 in the muon frame, what is its *velocity vector* in the lab-frame? Show entire calculation (**4 points**)

Use Einstein velocity transformation formula. We have $\beta=0.94$ from (b). Further we have $v'x=0$, $v'y=0.5$, $v'z=0$ (or v'z instead of v'y; both OK). $vx = \beta = 0.94$ **[1pnt for formula+number; ½pnt if only nr or only formula]** $vy = v'y.\sqrt{(1-\beta^2)} = v'y/\gamma = 0.5/2.93 = 0.17$ **[same]** $vz = v'z=0$ **[same]** **V**= (0.94, 0.17, 0) **[1 pnt for giving full vector]**

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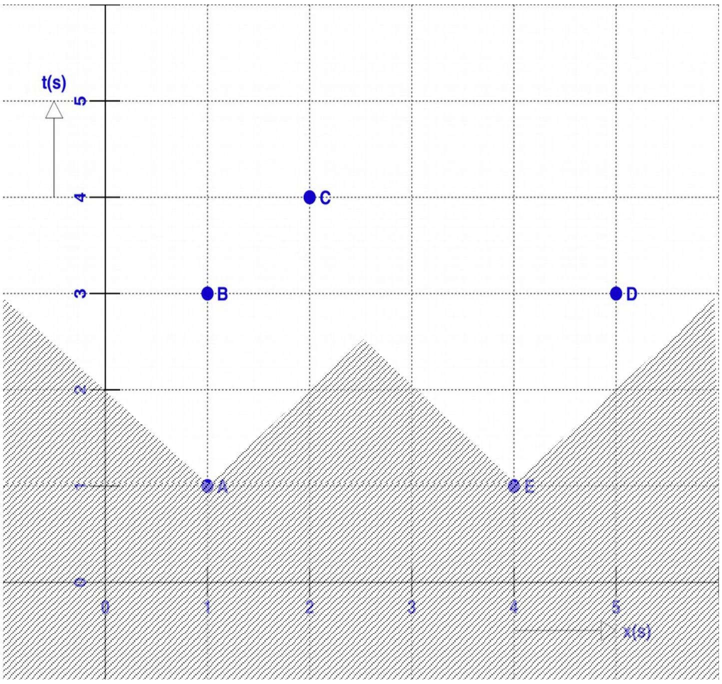
STUDENT NUMBER: s……………..…………………………...……………………………

Problem 2 – Who could have done it? (12 points)



(a) For each of the ten event-pairs in the spacetime diagram above, indicate whether the spacetime interval between them is timelike (**T**), lightlike (**L**), or spacelike (**S**). (**5 points**)

AB:	**T**	BC:	**L**	CD:	**S**	DE:	**T**
AC:	**T**	BD:	**S**	CE:	**T**		
AD:	**S**	BE:	**S**				
AE:	**S**						

(b) Which event(s) can cause event **C**? Tick all that apply. (**4 points**) A: X B: X D: □ E: X

[–2 points per wrong answer, min:0]

(c) In the diagram above shade the area(s) that can NOT be caused by *any* of the events A–E. Stay within the diagram. (**3 points**)

[Area below line through A–E: 1 point; area left/45deg from A & right/45deg from E: 1 point; area between A&E @ 45deg: 1 point]

NAME: ……………………………..……………………………..……………………………

STUDENT NUMBER: s……………..……………………………...……………………………

Problem 3 – Dyson sphere (10 points)

A Dyson sphere is a hypothetical megastructure that completely encompasses a star and captures all its power output. Use dimensional analysis to find out how the amount of harvested energy **E** during some time-period **P** depends on the mass **M**, volume **V** and surface temperature **T** of the star. From Earth-based experiments we know that **E** is proportional to the Stefan-Boltzmann constant **σ**, which has units $J \cdot s^{-1} \cdot m^{-2} \cdot K^{-4}$, so **E**= **σ**·f(**P**,**M**,**V**,**T**). Recall E= ½mv².

Assume that $\mathbf{E}=\boldsymbol{\sigma}\cdot f(\mathbf{P},\mathbf{M},\mathbf{V},\mathbf{T})=\boldsymbol{\sigma}\cdot \mathbf{P}^{i}\cdot \mathbf{M}^{j}\cdot \mathbf{V}^{k}\cdot \mathbf{T}^{l}$

J/Joule is the unit of energy, $E=\frac{1}{2} mv^2$, so $[E]=[M]([L]/[T])^2$, 1 J=1 $kg.m^2/s^2$

Find dimensions of left- and righthandside of equation

$[M][L]^2[T]^{-2} = [M][L]^2[T]^{-2}[T]^{-1}[L]^{-2}[\Theta]^{-4} \cdot [T]^i \cdot [M]^j \cdot [L]^{3k} \cdot [\Theta]^l$ **[2pnt]**

Gather powers of equal dimensions left and right

$[M]^1[L]^2[T]^{-2}[\Theta]^0=[M]^{j+1}[L]^{3k-2+2}[T]^{i-1-2}[\Theta]^{l-4}$ **[2pnt]**

Equate powers left and right

1=j+1 ; 2=3k ; -2=i-3 ; 0 = l-4 **[2pnt]**

Solve equations

j=0 ; k =2/3; i=1 ; l=4 **[2pnt]**

$\mathbf{E}=\boldsymbol{\sigma}\cdot \mathbf{P}\cdot \mathbf{V}^{2/3}\cdot \mathbf{T}^{4}$ **[2pnt]**

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STUDENT NUMBER: s……………..……………………………...……………………………

Problem 4 – Spaceport *"Edlee"* (10 points)

Spaceport *"Edlee"* has opened its first terminal for high-speed inter-planetary space ships. To guide the pilots during the landing of the space ships, a new landing strip is lined with blinking lights that are placed 22.5 m apart. The light placed at the beginning of the strip (x=0) blinks first (at t=0), the next one blinks 25 ns after the first, again 25 ns later the third one blinks, and so on. To avoid collisions, approaching pilots are told to "follow the lights" so that they all land in the same direction.

(a) In the spacetime diagram on the next page indicate events A–E for one sequence of flashes of the five lights at the beginning of the landing strip (in the spaceport frame). Make sure that the diagram can be read unambiguously. (**4 points**) **[units +1, nrs +1, events +2]**

(b) Captain Amandla is about to land her *Scintilla-50* spaceship (typical cruising speed 0.5 *c*) when ground control instructs her to abort the landing because she is approaching from the wrong direction. She claims to have followed instructions and that ground control must be mistaken, which they deny. Show how both could be correct using the spacetime diagram and a brief explanation. (**4 points)**

The spacetime interval between two flashes is **spacelike**, with v=Δx/Δt=3; so there is a velocity (v>1/3) for which the events A–E are observed in reversed time ordering.
So if Amandla was approaching with v>1/3 she would see the events occuring in reversed time order, and thus approach from the wrong direction *in the lab frame*. In the diagram this could by shown by drawing the x'-axis through A – E or one steeper than that for larger beta (e.g. 0.5 as suggested in the question). **Essence: $\Delta s^2<0$ interval → t-order flips**

(c) Which change to the *guide lights* would *guarantee* that the pilots land in the right direction? Very briefly explain. (**2 points)**

The interval between two successive flashes should be **timelike** to avoid time-reversal. However, in the case of a timelike interval the spatial order can be reversed. Hence the only solution would be lightlike. Approaching from one direction would cause all the lights to flash simultaneously. Spacelike leads to confusion.
Any reasonable argument resolving the t-reversal is acceptable

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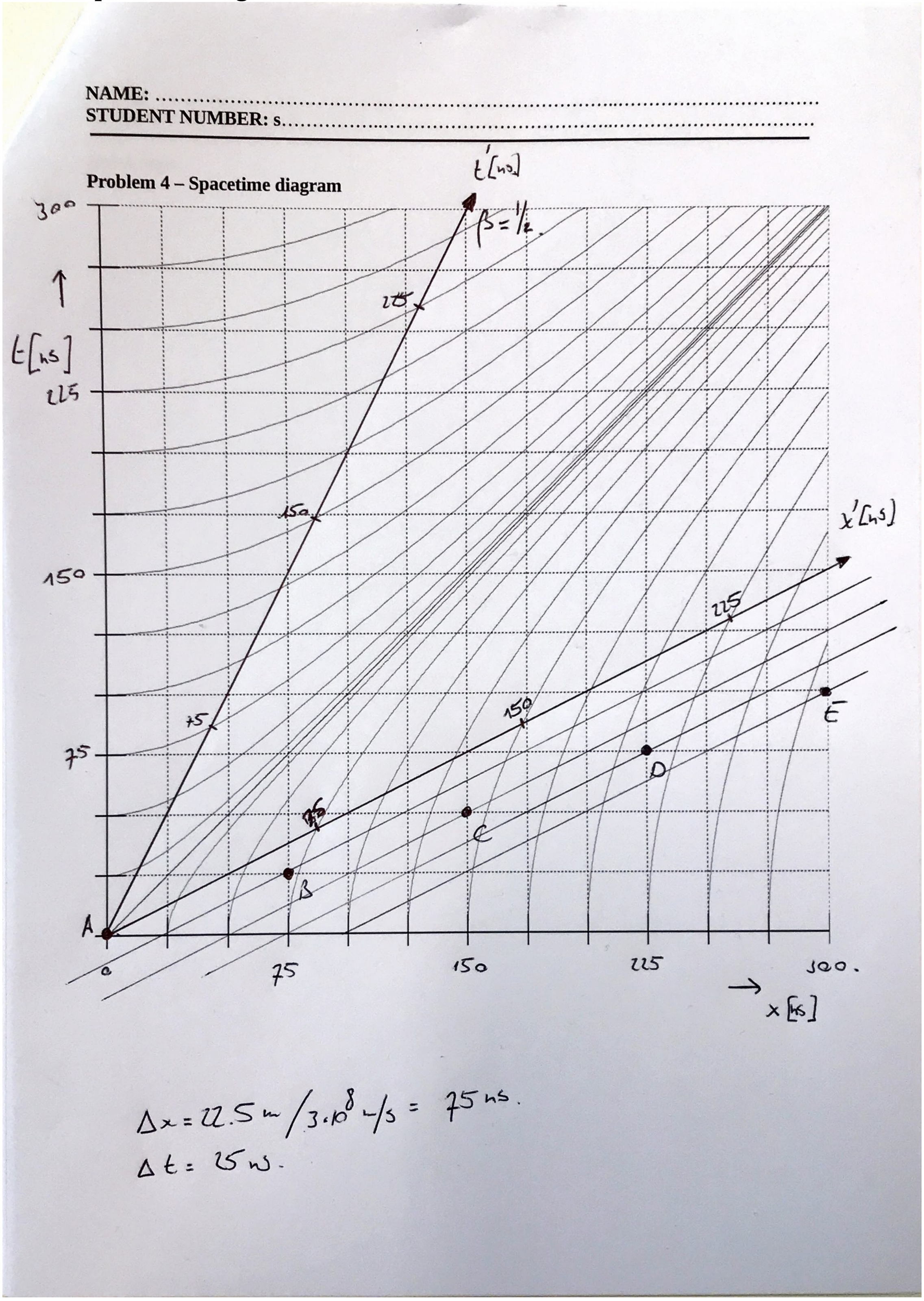
STUDENT NUMBER: s……………..………………………………..……………………………

Problem 4 – Spacetime diagram



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